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STABILITY OF STRATIFIED ELASTO-VISCOUS WALTERS' (MODEL B′) FLUID IN THE PRESENCE OF TWO - DIMENSIONAL HORIZONTAL MAGNETIC FIELD

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ABSTRACT

The effect of 2-D horizontal magnetic field is considered on the stability of stratified viscoelastic Walters' (Model B′) fluid in the presence of rotation. In contrast to the Newtonian fluids, the system is found to be unstable for small wavelength perturbations for the case of stable stratification. It is also found that the magnetic field stabilizes the certain wave-number band for unstable stratification in the presence of rotation and these wave-number range increases with the increase in magnetic field and decreases with the increase in kinematic visco-elasticity.

INTRODUCTION:

The character of the equilibrium of an inviscid, incompressible fluid having variable density in the vertical direction was investigated by RAYLEIGH [1]. He demonstrated that the system is stable or unstable according as the density decreases everywhere or increases anywhere. A comprehensive account of the Rayleigh–Taylor instability was given by CHANDRASEKHAR [2] wherein the effects of uniform rotation with an angular velocity $\vec{\Omega}$ about the vertical and uniform horizontal magnetic field, separately, were also treated. REID [3] studied the effects of surface tension and viscosity on the stability of two superposed fluids. BELLMAN and PENNINGTION [4] investigated in detail the combined effects of viscosity and surface tension on Taylor instability. GUPTA [5] again studied the stability of a horizontal layer of an electrically conducting fluid with continuous density and viscosity stratification in the presence of a horizontal magnetic field. The effect of a vertical magnetic field on the development of the Rayleigh–Taylor instability was considered by HIDE [6]. BHATIA and SHARMA [7] studied the Rayleigh Taylor instability of rotating stratified fluid in an inhomogeneous magnetic field. Generally the magnetic field has a stabilizing effect on the instability, but there are also a few exceptions. For example, KENT [8] has studied the effect of horizontal magnetic field which varies in the vertical direction on the stability of parallel flows and has shown that the system is unstable under certain conditions, while in the absence of magnetic field the system is known to be stable. In all the above studies, the fluid has been assumed to be Newtonian. With the growing importance of non-Newtonian fluids in modern technology and industries, the investigation of such fluids is desirable. There are many elasto-viscous fluids that cannot be characterized by Maxwell’s or Oldroyd’s constitutive relations. One such class of fluids is Walters’ (Model B′) fluid. WALTERS [9] proposed a theoretical model for such elasto-viscous fluids. Many other research workers have paid their attention to the study of Walters’ (Model B′) fluids. The behaviour of the mixture of polymethyl methacrylate and pyridine at 25 °C containing 30.5 grams of polymer per litre is very similar to that of Walters’ (Model B′) viscoelastic fluid (WALTERS [10]). The fluids of this class are used in the manufacture of the parts of space craft, aeroplanes, tyres, belt conveyors, ropes, cushions, seats, foams, plastics, engineering equipments, etc.

SHARMA and KUMAR [11] studied the effects of the presence of a transverse magnetic field on the stability of two superposed Walters' (Model B′) viscoelastic liquids. SHARMA and KUMAR [12] also studied the Rayleigh–Taylor instability of stratified Walters' (Model B′) fluid in the presence of a variable horizontal magnetic field and suspended particles. The Coriolis force also affects significantly the stability of geophysical phenomenon. Keeping in mind the conflicting tendencies of magnetic field and rotation while acting together, we set out to study the combined effect of two dimensional magnetic field and rotation on the stability of stratified elasto-viscous Walters' (Model B′) fluid. The same problem for one dimensional horizontal magnetic field studied by SHARMA and GUPTA [13].

FORMULATION OF THE PROBLEM AND PERTURBATION EQUATIONS:

The initial stationary state, whose stability we wish to examine, is that of an incompressible, heterogeneous infinitely extending elasto-viscous Walters' (Model B′) fluid of variable density, kinematic viscosity and kinematic viscoelasticity so that the free surface is almost horizontal. The fluid is acting by the gravity force $\vec{g}$ (0, 0, $-g$),a uniform horizontal rotation $\vec{\Omega}$ (Ω, 0, 0) and a uniform 2- dimensional horizontal magnetic field $\vec{H}$ (H_x, H_y, 0). The character of the equilibrium of this stationary state can be determined by disturbing the system slightly and then, following its further evolution. Let ρ, μ, μ′, p and $\vec{u}$ (u, v, w) denote, respectively, the density, the viscosity, the viscoelasticity, the pressure and the velocity of fluid (initially zero). Then the equations expressing the conservation of momentum, mass, incompressibility and Maxwell's equation for the elasto-viscous Walters' (Model B′) fluid are

$$\rho\left[\frac{\partial\vec{u}}{\partial t}+(\vec{u}.\nabla)\vec{u}\right]=-\nabla p+\vec{g}\rho+\left(\mu-\mu'\frac{\partial}{\partial t}\right)\nabla^2\vec{u}+2\rho(\vec{u}\times\vec{\Omega})+\frac{\mu_e}{4\pi}\left[\left(\nabla\times\vec{H}\right)\times\vec{H}\right],\ldots(1)$$

$$\nabla.\vec{u}=0 \qquad \ldots(2)$$

$$\frac{\partial\rho}{\partial t}+(\vec{u}.\nabla)\rho=0 \qquad \ldots(3)$$

$$\nabla.\vec{H}=0, \qquad \ldots(4)$$

$$\frac{\partial \vec{H}}{\partial t} = \nabla \times (\vec{u} \times \vec{H}), \qquad \ldots(5)$$

where μ_e, the magnetic permeability, is assumed to be constant.

Equation (3) represents the fact that the density of a particle remains unchanged as we follow it with its motion.

Let $\delta\rho$, δp, $\vec{u}$ (u, v, w) and $\vec{h}$ (h_x, h_y, h_z) denote, respectively, the perturbations in the density $\rho(z)$, the pressure $p(z)$, the velocity $\vec{u}$ (0, 0, 0) and the horizontal magnetic field $\vec{H}$ $(H_x, H_y, 0)$.

Then the linearized perturbation equations become

$$\rho \frac{\partial \vec{u}}{\partial t} = -\nabla \delta p + \vec{g}\delta\rho + \left(\mu - \mu' \frac{\partial}{\partial t}\right)\nabla^2 \vec{u} + 2\rho(\vec{u} \times \vec{\Omega}) + \frac{\mu_e}{4\pi}\left[(\nabla \times \vec{h}) \times \vec{H}\right] \ldots(6)$$

$$\nabla . \vec{u} = 0 \qquad \ldots(7)$$

$$\frac{\partial}{\partial t}(\delta\rho) + (\vec{u}.\nabla)\rho = 0 \qquad \ldots(8)$$

$$\nabla . \vec{h} = 0, \qquad \ldots(9)$$

$$\frac{\partial \vec{h}}{\partial t} = \nabla \times (\vec{u} \times \vec{H}), \qquad \ldots(10)$$

Analyzing the disturbances in normal modes, we seek the solutions whose dependence on x, y, z and time t is given by

$$f(z) \exp(ik_x x + ik_y y + nt), \qquad \ldots(11)$$

where $f(z)$ is some function of z; k_x, k_y are the wave numbers along x and y axes, $k = (k_x^2 + k_y^2)^{1/2}$ is the resultant wave number and n (may be complex) denotes the rate at which system departs from equilibrium.

Equations (6)–(10), using expression (11), in the Cartesian coordinates become

$$\rho n u = -ik_x \delta p + (\mu - \mu' n)(D^2 - k^2)u, \qquad \ldots(12)$$

$$\rho n v = -ik_y \delta p + (\mu - \mu' n)(D^2 - k^2)v$$

$$+ \frac{\mu_e H}{4\pi}(ik_x h_y - ik_y h_x) + 2\rho \Omega w \qquad \ldots(13)$$

$$\rho n w = -D\delta p + (\mu - \mu' n)(D^2 - k^2)w$$

$$+ \frac{\mu_e H}{4\pi}(ik_x h_y - Dh_x) - 2\rho\Omega v - g\delta p, \qquad \ldots(14)$$

$$ik_x u + ik_y v + Dw = 0, \qquad \ldots(15)$$

$$n\delta\rho + w(D\rho) = 0, \qquad \ldots(16)$$

$$ik_x h_x + ik_y h_y + Dh_z = 0 \qquad \ldots(17)$$

$nh_x = ik_x Hu,$ …(18)

$nh_y = ik_x Hv$ …(19)

$nh_z = ik_x Hw$ …(20)

where D stands for d/dz.

Eliminating some of the variables from equations (12) – (14) and using equations (15) – (20), we obtain an equation in terms of w as

$$\rho n(\nu + \nu' n)(D^2 - k^2)^2 w - \rho[n^2 + k_x^2 V_A^2](D^2 - k^2)w - n^2(D\rho)(Dw)$$

$$+ \left[\frac{4\rho n^2 \Omega^2 k_x^2}{n^2 - n(\nu + \nu' n)(D^2 - k^2) + k_x^2 V_A^2} - gk^2(D\rho)\right]w - 2ink_y\Omega(D\rho)w = 0$$

…(21)

where $\nu = \mu/\rho$, $\nu' = \mu'/\rho$ and $V_A^2 = \mu_e H^2 / 4\pi\rho$ (square of the Alfve′n velocity).

EXPONENTIALLY VARYING STRATIFICATIONS:

Let us assume the stratifications in density, viscosity, viscoelasticity of the forms

$$\rho = \rho_0 e^{\beta z},\ \mu = \mu_0 e^{\beta z}\ ,\mu' = \mu_0' e^{\beta z} \qquad \ldots(22)$$

where ρ_0 , μ_0 , μ'_0 and β are constants, and therefore, the kinematic viscosity

$$v_0\left(= \frac{\mu}{\rho} = \frac{\mu_0}{\rho_0}\right)$$

and kinematic viscoelasticity

$$v'_0\left(= \frac{\mu'}{\rho} = \frac{\mu'_0}{\rho_0}\right)$$

are constant everywhere.

Using stratifications of the form (22), equation (21) transforms to

$$\left(D^2 - k^2\right)^3 w - \frac{2}{n\left(v_0 - v_0' n\right)}\left[\left(n^2 + k_x^2 V_A^2\right)\right]\left(D^2 - k^2\right)^2 w$$

$$+ \frac{1}{n^2\left(v_0 - v_0' n\right)^2}\left[n^4 + k_x^2 V_A^2\left(2n^2 + k_x^2 V_A^2\right) - g\beta k^2 n\left(v_0 - v_0' n\right)\right]\left(D^2 - k^2\right) w$$

$$- \frac{1}{n^2\left(v_0 - v_0' n\right)^2}\left[4n^2 k_x^2 \Omega - g\beta k^2 n\left(n^2 + k_x^2 V_A^2\right)\right] w = 0 \qquad \ldots (23)$$

Considering the case of two free boundaries, we must have

$w = D^2 w = 0$ at $z = 0$ and $z = d$. …(24)

The appropriate solution of equation (23) satisfying the above boundary conditions is

$$w = A \sin \frac{m\pi z}{d} \qquad \ldots (25)$$

where m is an integer and A is a constant.

Inserting the value of w from equation (25) into equation (23), we obtain the dispersion relation

$$n^4\left[\left(1-v_0^{'}L_1\right)^2\right]+n^3\left[2v_0L_1\left(1-v_0^{'}L_1\right)\right]+n^2\left[v_0^2L_1^2+\left(2k_x^2V_A^2-\frac{g\beta k^2}{L_1}\right)\right.$$

$$\left.\left(1-v_0^{'}L_1\right)+\frac{4k_x^2\Omega^2}{L_1}\right]+n\left[v_0L_1\left(2k_x^2V_A^2-\frac{g\beta k^2}{L_1}\right)\right]+k_x^2V_A^2\left(k_x^2V_A^2-\frac{g\beta k^2}{L_1}\right)=0$$

…(26)

where

$$L_1=\left[k^2+\left(\frac{m\pi}{d}\right)^2\right]$$

Equation (26) is biquadratic in n, therefore, it must give four roots, and is the dispersion relation representing the effects of rotation and horizontal magnetic field on the stability of stratified (exponentially varying density) elasto-viscous Walters' (Model B′) fluid in non-porous medium.

RESULTS AND DISCUSSION:

(a) Case of stable stratification (i.e. $\beta < 0$). If $\beta < 0$ and) 1 (1ν'_0L$_1$) > 0, then equation (26) does not admit of any positive real root nor complex root with positive real part and, therefore, the system is stable for disturbances of all wave-numbers. However, it is clear that the system is unstable for) 1 1 (1ν'_0L$_1$) < 0.

Thus for stable stratification, the system is stable for disturbances of all wave-numbers satisfying

$$k^2<\left(\frac{1}{v_0^{'}}-\frac{m^2\pi^2}{d^2}\right), \qquad \ldots(27)$$

and unstable otherwise. Thus the system becomes destabilized for small wavelength perturbations even if it is a bottom heavy (stable) configuration. This stands in contrast to the Newtonian fluids where the system is always stable for stable stratification (CHANDRASEKHAR [2]). The condition for the system to be unstable is

$(1-\nu'_0 L_1) < 0$

i.e.

$$\nu'_0 > \frac{d^2}{k^2 d^2 + m^2 \pi^2} \qquad \ldots(28)$$

Numerical model example. For the depth of the fluid layer d = 6 cm, the wave number k = 0.2, the integer m = 1, the condition for instability

$$\nu'_0 > \frac{d^2}{k^2 d^2 + m^2 \pi^2}$$

gives $\nu_0' > 3.6$, i.e. the fluid layer for the case of stable stratification will be unstable if kinematic viscoelasticity of the fluid will be greater than 3.6 cm^2/sec. Similarly, for d= 6 cm, k = 0.5, m = 1, the condition for instability leads to $\nu_0' > 2.0$; and for d = 6 cm, k = 1.0, m = 1, it gives $\nu_0' > 1.0$ Thus, we have seen that as viscoelasticity increases a wider range of wave numbers become unstable.

We have examined the behaviour of growth rates with respect to the kinematic viscoelasticity ν_0' satisfying equation (26) numerically for the case of stable stratification. Figure 1 shows the variation of the growth rate nr (positive real value of n) with respect to the wave number k satisfying equation (26), for the fixed permissible values of $\beta = -2$, m = 1, d = 6 cm, ν_0 = 4, Ω = 6 rotations/min, g = 980 cm/sec^2, $V_A^2 = 15$, $kx = k\cos 45°$, for five values of ν_0' = 0.5, 1.0, 2.0, 3.0 and 4.0, respectively, for the wavenumber range, $2 \le k \le 1.6$. The plots show that kinematic viscoelasticity ν_0' has a destabilizing effect on the system and as viscoelasticity increases a wider range of wave numbers become unstable. It is clear from figure1 that for ν_0' = 3 and 4, the system is unstable for the whole range of wave numbers. For ν_0' = 2, the system is unstable for $k > 0.8$ and finally for ν_0' = 0.5, the system is unstable for $k > 1.2$. Thus figure 1 confirms the earlier result that it is an increase in the viscoelasticity that increases the wave-number band which is unstable.

In figure 2, we have plotted the variation of nr (positive real value of n) for the same set of various parameters for the wave number range $1 \le k \le 4$. It is clear from figure 2 that as the wave number increases, the system is unstable even for lesser viscoelastic fluids which confirms the analytic result drawn earlier that the system gets destabilized for small wavelength perturbations.

(b) Case of unstable stratification (i.e. β > 0). Based on equation (26) it can be inferred that if

$$V_A^2 < \frac{g\beta k^2}{\left(k^2 + \frac{m^2\pi^2}{d^2}\right)k_x^2}, \qquad \ldots(29)$$

the constant term is negative and, therefore, has at least one positive real root. Hence the system is unstable for all wave numbers satisfying the inequality

$$k^2 < \frac{g\beta \sec^2\Phi}{V_A^2} - \frac{m^2\pi^2}{d^2}, \qquad \ldots(30)$$

where Φ is the angle between *kx* and *k* (i.e. *kx* = *k*cos Φ). However if

$$\left(1 - v_0^{'}L_1\right) > 0 \text{ and } \quad V_A^2 < \frac{g\beta k^2}{\left(k^2 + \frac{m^2\pi^2}{d^2}\right)k_x^2}, \qquad \ldots(31)$$

equation (26) does not admit of any positive real root nor complex root with positive real part and therefore the system is stable. The magnetic field, therefore, stabilizes potentially unstable stratification for the wave-number band

$$\frac{g\beta \sec^2\Phi}{V_A^2} < k^2 + \frac{m^2\pi^2}{d^2} < \frac{1}{v_0^{'}} \qquad \ldots (32)$$

Also, it is clear that the wave-number range, for which the potentially unstable system gets stabilized, increases with the increase in magnetic field and decreases with the increase in kinematic viscoelasticity. All long wavelength perturbations satisfying (30) remain unstable and are not stabilized by magnetic field.

We have examined the behaviour of growth rates with respect to the kinematic viscosity ν and the kinematic viscoelasticity ν′ satisfying equation (26) numerically. Figure 3 shows the variation of the growth rate *nr* (positive real value of *n*) with respect to the wave number k satisfying equation (26), for the fixed permissible values of $\beta = 2$, $m = 1$, $d = 6$ cm, $\nu_0 = 4$, $\Omega = 6$ rotations/min, $g = 980$ cm/sec^2, $V_A^2 = 15$, $k_x = k\cos45°$; for four values of $\nu_0' = 0.5$, 1.0, 1.5 and 2.0, respectively, for the wavenumber range $0.2 \leq k \leq 1.6$. The graph shows that the kinematic viscoelasticity ν_0' has a destabilizing effect for low wave numbers and this destabilizing influence increases with the increase in the kinematic viscoelasticity ν_0' for low wave numbers. This supports our conclusions drawn mathematically that the long wavelength perturbations remain unstable. However, as the wave-number range increases, the system gets stabilized with the increase in kinematic viscoelasticity ν_0' as is evident from figure 4 for the wave-number range $1 \leq k \leq 4$.

Figure 5 shows the variation of the growth rate *nr* (positive real value of *n*) with respect to the wave number k for the fixed permissible values of $\beta = 2$, $m = 1$, $d = 6$ cm, $V_0' = 1$, $\Omega = 6$ rotations/min, $g = 980$ cm/sec^2, $V_A^2 = 15$, $k_x = k\cos45°$; for three values of $\nu_0 = 2$, 4 and 6, respectively, for the wave-number range $0.2 \leq k \leq 1.2$. The graph shows that kinematic viscosity ν_0 has a stabilizing effect for the low wave number range with the increase in kinematic viscosity. However, as the wave number range increases, the the kinematic viscosity has a destabilizing effect with the increase in kinematic viscosity as is clear from figure 6 for the wave-number range $2 \leq k \leq 4$.

CONCLUSIONS:

The principle conclusions drawn from the analysis of the present paper are as follows:

(i) In contrast to the Newtonian fluids, the system gets destabilized for Walters' (Model B′) fluid for small wavelength perturbations even if it is a bottom heavy configuration.

(ii) For stable stratification, as the viscoelasticity increases, the wave-number band for which the system becomes unstable increases.

(iii) Magnetic field stabilizes certain wave-number range and this range increases with the increase in magnetic field.

(iv) The long wavelength perturbations remain unstable (for potentially unstable stratification) and are not stabilized by magnetic field.

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