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STABILITY OF INCOMPRESSIBLE RIVLIN-ERICKSEN ELASTICO-VISCOUS SUPERPOSED FLUIDS PERMEATED WITH SUSPENDED PARTICLES AND VARIABLE HORIZONTAL MAGNETIC FIELD IN POROUS MEDIUM

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ABSTRACT

The stability of incompressible Rivlin-Ericksen elastico-viscous superposed fluids permeated with suspended particles and variable horizontal magnetic field in porous medium has been investigated. By applying normal mode analysis method, the dispersion relation has been derived and solved numerically. The case of exponential varying stratifications has been discussed and it has been found that the system is stable for stable stratifications if $\beta < 0$ and unstable for unstable stratifications if $\beta > 0$. The variable horizontal magnetic field stabilizes the system for certain wave number range. The growth rates decrease with the increase in kinematic viscosity, kinematic viscoelasticity, suspended particles number density and magnetic field whereas growth rates increase with the increase in medium permeability.

Keywords:

Rivlin-Ericksen elastico-viscous superposed fluid, magnetic field, suspended particles, porous medium

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NOMENCLATURE

g	Acceleration due to gravity
H	Magnetic field
m	Mass of suspended particles
k_1	Medium permeability
p	Pressure
K'	Stokes drag coefficient
N	Suspended particles number density

Greek Symbols

$\boldsymbol{\rho}$	Fluid density
υ	Kinematic viscosity
υ'	Kinematic viscoelasticity
μ_e	Magnetic permeability
$\in$	Medium porosity
η	Particle radius
μ	Viscosity of fluid
μ'	Viscoelasticity of fluid

INTRODUCTION

The character of equilibrium of an inviscid, incompressible fluid having variable density in the vertical direction has been investigated by RAYLEIGH [1]. He demonstrated that the system is stable or unstable according as the density decreases everywhere or increases anywhere. A comprehensive account of the Rayleigh-Taylor instability under varying assumptions of hydrodynamics and hydromagnetics has been given by CHANDRASEKHAR [2]. OLDROYD [3] has studied some steady flows of a general elastico-viscous liquid while the effect of surface tension and viscosity on the stability of two superposed fluids has been studied by RIED [4].

The effects of suspended particles on the stability of stratified fluids find importance in geophysics and chemical engineering. Further, motivation for stability of fluids in the presence of suspended particles is the fact that knowledge concerning fluid-particle mixtures is not commensurate with their industrial and scientific importance. SCANLON and SEGAL [5] have considered the effect of suspended particles on the onset of Be'nard convection and found that the critical Rayleigh number was reduced solely because the heat capacity of the pure gas was supplemented by the particles. The suspended particles were thus found to destabilize the layer.

The medium has been considered to be non-porous in all the above studies. WOODING [6] has considered the Rayleigh instability of thermal boundary layer in flow through porous medium whereas SHARMA [7] has studied thermal instability of a viscoelastic fluid in hydromagnetics. SHARMA and SHARMA [8] have also studied hydromagnetic instability of streaming fluid in porous medium. With the growing importance non-Newtonian fluids in modern technology and industries, the investigations on such fluids are desirable. There are many elastic-viscous fluids that cannot be characterized by Maxwell's constitutive relations or Oldroyd's constitutive relations. One such class of elastic-viscous fluid is Rivlin-Ericksen fluid. RIVLIN and ERICKSEN [9] have proposed a theoretical model for such another elastico-viscous fluid.

The stability of flow of fluid through a porous medium taking in to account by LAPWOOD [10].

A porous medium is a solid with holes in it, and is characterized by the manner in which the holes are imbedded, how they are interconnected and the description of their location, shape and interconnection. However, the flow of a fluid through a homogeneous and isotropic porous medium is governed by Darcy's law which states that the usual viscous term in the equations of stratified Rivlin-Ericksen fluid motion is replaced by the resistance term $\left[-\frac{1}{k_1}\left(\mu+\mu'\frac{\partial}{\partial t}\right)\right]q$, where μ and μ' are the viscosity and viscoelasticity of the incompressible Rivlin-Ericksen fluid, k_1 is the medium permeability and q is the Darcian (filter) velocity of the fluid.

SHARMA and RANA [11] have studied the instability of streaming Rivlin-Ericksen fluids in Porous Medium in hydromagnetics whereas SHARMA et al. [12] have studied the instability of streaming Rivlin-Ericksen fluids in porous medium. Recently, RANA et al. [13] have studied stability of incompressible Rivlin-Ericksen elastic-viscous superposed fluids in the presence of rotation in porous medium. Keeping in mind the importance in various applications mentioned above, our interest, in the present paper is to study the stability of incompressible Rivlin-Ericksen elastico-viscous superposed fluids permeated with suspended particles and variable horizontal magnetic field in a porous medium.

FORMULATION OF THE PROBLEM AND PERTURBATION EQUATIONS

The initial stationary state whose stability we wish to examine is that of an incompressible Rivlin-Ericksen elastico-viscous superposed fluid of variable density, kinematic viscosity and kinematic viscoelasticity permeated with suspended particles and variable horizontal magnetic field arranged in horizontal strata in porous medium of variable porosity and permeability. The character of equilibrium of this initially static state is determined by supposing that the system is slightly disturbed and then following its further evolution.

Let $\rho\,,\upsilon\,,\upsilon^{'},p,\in$ and q (0, 0, 0), denote, respectively, the density, the kinematic viscosity, the pressure, the porosity and velocity of the fluid. The fluid is acted upon by a uniform horizontal magnetic field $H(H_0(z),0,0)$ and gravity force g (0, 0,-g). Then the equations of motion, continuity, incompressibility and Maxwell's equations for the incompressible Rivlin-Ericksen elastico-viscous superposed fluid permeated with suspended particles and uniform horizontal magnetic field through a porous medium are

$$\frac{\rho}{\in}\left[\frac{\partial q}{\partial t}+\frac{1}{\in}(q.\nabla)q\right]=-\nabla p+g\rho-\frac{\rho}{k_1}\left(\upsilon+\upsilon'\frac{\partial}{\partial t}\right)q+\frac{\mu_e}{4\pi}(\nabla\times H)\times H+\frac{K'N}{\in}(q_d-q), \quad (1)$$

$$\nabla.q=0, \quad (2)$$

$$\in\frac{\partial\rho}{\partial t}+(q.\nabla)\rho=0, \quad (3)$$

$$\nabla.H=0, \quad (4)$$

$$\in\frac{\partial H}{\partial t}=\nabla\times(q\times H), \quad (5)$$

where μ_e stands for magnetic permeability.

Here $q_d(\bar{x},t)$ and $N(\bar{x},t)$ denote the velocity and number density of the particles respectively,$K^{'}=6\pi\eta\rho\upsilon$, where η is particle radius, is the Stokes drag coefficient, $q_d=(l,r,s)$ and $\bar{x}=(x,y,z)$.

If mN is the mass of particles per unit volume, then the equations of motion and continuity for the particles are

$$mN\left[\frac{\partial q_d}{\partial t}+\frac{1}{\in}(q.\nabla)q_d\right]=K'N(q-q_d), \quad (6)$$

$$\in\frac{\partial N}{\partial t}+\nabla.(Nq_d)=0. \quad (7)$$

The presence of suspended particles adds an extra force term proportional to the velocity difference between particles and fluid and appears in the equation of motion (1). Since the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equations of motion for the particles (6). The buoyancy force on the particles is neglected. Interparticles reactions are not considered either since we assume that the distance between the particles are quite large compared with their diameters. These assumptions have been used in writing the equations of motion (6) for the particles.

Let q (u, v, w), h (h_x, h_y, h_z), δp and $\delta\rho$ denote, respectively, the perturbations in the velocity q (0, 0, 0), the perturbation in magnetic field H ($H_0(z)$, 0, 0), the pressure p, the density ρ. The linearized perturbation equations governing the motion of fluids are

$$\frac{\rho}{\epsilon}\left[\frac{\partial}{\partial t}+\frac{\epsilon}{k_1}\left(\upsilon+\upsilon'\frac{\partial}{\partial t}\right)\right]q=-\nabla\delta p+g\delta\rho+\frac{\mu_e}{4\pi}[(\nabla\times h)\times H+(\nabla\times H)\times h]+\frac{K'N}{\epsilon}(q_d-q) \quad (8)$$

$$\nabla.q=0\,, \quad (9)$$

$$\epsilon\frac{\partial(\delta\rho)}{\partial t}=-\frac{d\rho}{dz}. \quad (10)$$

$$\nabla.h=0, \quad (11)$$

$$\epsilon\frac{\partial h}{\partial t}=(H.\nabla)q-(q.\nabla)H, \quad (12)$$

$$\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)q_d=q. \quad (13)$$

THE DISPERSION RELATION

Following the normal mode analyses, we assume that the perturbation quantities have x, y and t dependence of the form

$$exp\left(ik_x x+ik_y y+nt\right), \quad (14)$$

where k_x and k_y are the wave numbers in the x and y directions, $k=\left(k_x{}^2+k_y{}^2\right)^{1/2}$ is the resultant wave number and n is the frequency of the harmonic disturbance, which is, in general, a complex constant. For solutions having this dependence on x, y and t, equations (6)-(13) using equation (14) in the Cartesian coordinates become

$$\frac{\rho}{\epsilon}\left[n+\frac{\epsilon}{k_1}\left(\upsilon+\upsilon' n\right)+\frac{mNn/\rho}{mn/K'+1}\right]u=-ik_x\delta p+\frac{\mu_e}{4\pi}h_z DH_0, \quad (15)$$

$$\frac{\rho}{\epsilon}\left[n+\frac{\epsilon}{k_1}\left(\upsilon+\upsilon' n\right)+\frac{mNn/\rho}{mn/K'+1}\right]v=-ik_y\delta p+\frac{\mu_e}{4\pi}(ik_x h_y-ik_y h_x), \quad (16)$$

$$\frac{\rho}{\epsilon}\left[n+\frac{\epsilon}{k_1}\left(\upsilon+\upsilon' n\right)+\frac{mNn/\rho}{mn/K'+1}\right]w=-D\delta p-g\delta\rho+\frac{\mu_e}{4\pi}\left(ik_x h_z-Dh_x-h_x\frac{DH_0}{H_0}\right), \quad (17)$$

$$ik_x u+ik_y v+Dw=0, \quad (18)$$

$$\in n\delta\rho = -wD\rho, \tag{19}$$

$$ik_x h_x + ik_y h_y + Dh_z = 0, \tag{20}$$

$$\in nh_x = ik_x H_0 u - wDH_0, \tag{21}$$

$$\in nh_y = ik_x H_0 v, \tag{22}$$

$$\in nh_z = ik_x H_0 w. \tag{23}$$

Eliminating u, v, h_x, h_y, h_z and δp from equations (15)-(17) and using equations (18)-(23), after a little algebra, we get

$$D\left[\frac{\rho}{\in}\left\{n + \frac{\in}{k_1}(\upsilon + \upsilon' n) + \frac{mNn/\rho}{mn/_{K'}+1}\right\}Dw\right] - \frac{\rho}{\in}\left\{n + \frac{\in}{k_1}(\upsilon + \upsilon' n) + \frac{mNn/\rho}{mn/_{K'}+1}\right\}k^2 w + \frac{\mu_e k_x^2}{4\pi\in n}\left[H_0^2(D^2 - k^2)w + D(H_0^2)Dw - H_0^2 - \frac{H_0^2(Dw)(D\in)}{\in}\right] = -\frac{gw(D\rho)k^2}{\in n}. \tag{24}$$

Equation (24) is the dispersion relation governing the stability of incompressible Rivlin-Ericksen elastico-viscous superposed fluid permeated with suspended particles and variable horizontal magnetic field in porous medium.

THE CASE OF EXPONENTIALLY VARYING STRATIFICATIONS

Using stratifications in fluid density, viscosity, fluid viscoelasticity, medium porosity medium permeability, suspended particles number density and magnetic field of the forms

$$\rho = \rho_0 e^{\beta z}, \mu = \mu_0 e^{\beta z}, \mu' = \mu'_0 e^{\beta z}, \in = \in_0 e^{\beta z}, k_1 = k_{10} e^{\beta z}, N = N_0 e^{\beta z}, H_0^2 = H_1^2 e^{\beta z}, \tag{25}$$

where ρ_0, μ_0, μ'_0, $\in_0$, k_{10}, N_0, H_1^2 and β are constants and so kinematic viscosity $\upsilon = {}^{\mu}/_{\rho} = {}^{\mu_0}/_{\rho_0} = \upsilon_0$ kinematic viscoelasticity $\upsilon' = {}^{\mu'}/_{\rho} = {}^{\mu_0'}/_{\rho_0} = \upsilon_0'$ and the Alfven velocity

$V_A \left(= \sqrt{\frac{\mu_e H_0^2}{4\pi\rho}} = \sqrt{\frac{\mu_e H_1^2}{4\pi\rho_0}}\right)$ are constants everywhere. We also assume that $\beta d \ll 1$ i.e. the variation of density at two neighboring points in the velocity field which is much less than average density has a negligible effect on the velocity of the fluid.

The boundary conditions for the case of two surfaces are

$$w = D^2 w = 0 \text{ at } z = 0 \text{ and } z = d. \tag{26}$$

The proper solution of equation (24) satisfying equation (26) is

$$w = A_0 sin\frac{m_0 \pi z}{d} \qquad (27)$$

where A_0 is a constant and m_0 is any integer.

Using stratifications of the form (25), substituting equation (27) in equation (24) and neglecting the effect of heterogeneity, we get

$$\left[\left(\frac{m_0\pi}{d}\right)^2 + k^2\right] - \frac{gk^2\beta}{n\left[n+\frac{\epsilon}{k_1}(\upsilon+\upsilon' n)+\frac{mNn}{mn}\!/\!\rho\!/\!K'+1}+\frac{k_x{}^2 V_A{}^2}{n}\right]} = 0, \qquad (28)$$

which on simplification, becomes

$$\left(1+\frac{\epsilon_0\upsilon_0'}{k_{10}}\right)n^3 + \left[\frac{K'}{m}\left(1+\frac{\epsilon_0\upsilon_0'}{k_{10}}+\frac{\epsilon_0\upsilon_0}{k_{10}}+\frac{K'N_0}{\rho_0}\right)\right]n^2 + \left[\frac{K'}{m}\frac{\epsilon_0\upsilon_0}{k_{10}} + k_x{}^2 V_A{}^2 - \frac{g\beta k^2}{L}\right]n + \left[\left(k_x{}^2 V_A{}^2 - \frac{g\beta k^2}{L}\right)\frac{K'}{m}\right] = 0, \qquad (29)$$

where $L = \left(\frac{m_0\pi}{d}\right)^2 + k^2$.

If β < 0 (stable stratification), equation (29) does not admit any positive root of n and so the system is always stable for disturbances of all wave numbers. If β > 0 (unstable stratification), the system is stable/unstable if

$$k_x{}^2 V_A{}^2 > \frac{g\beta k^2}{L} \,/\, k_x{}^2 V_A{}^2 < \frac{g\beta k^2}{L}. \qquad (30)$$

The system is clearly unstable in the absence of a magnetic field. However, the system can be completely stabilized by large enough magnetic field as can be seen from equation (30), if

$$V_A{}^2 > \frac{g\beta k^2}{L k_x{}^2}.$$

Thus, if β > 0 and $k_x{}^2 V_A{}^2 < \frac{g\beta k^2}{L}$, equation (30) has at least one positive root. Let n_0 denote the positive root of equation (29). Then

$$\left(1+\frac{\epsilon_0\upsilon_0'}{k_{10}}\right)n_0{}^3 + \left[\frac{K'}{m}\left(1+\frac{\epsilon_0\upsilon_0'}{k_{10}}+\frac{\epsilon_0\upsilon_0}{k_{10}}+\frac{K'N_0}{\rho_0}\right)\right]n_0{}^2 + \left[\left[\frac{K'}{m}\frac{\epsilon_0\upsilon_0}{k_{10}} + k_x{}^2 V_A{}^2 - \frac{g\beta k^2}{L}\right]\right]n_0 + \left[\left(k_x{}^2 V_A{}^2 - \frac{g\beta k^2}{L}\right)\frac{K'}{m}\right] = 0\ . \qquad (31)$$

To find the role of kinematic viscosity, kinematic viscoelasticity, medium permeability, suspended particles number density and magnetic field on the growth rates of unstable modes, we examine the nature of $\frac{dn_0}{d\upsilon_0}, \frac{dn_0}{d\upsilon_0{}'}, \frac{dn_0}{dk_{10}}, \frac{dn_0}{dN_0}, \frac{dn_0}{dV_A}$ analytically:

$$\frac{dn_0}{d\upsilon_0} = -\frac{\epsilon_0 n_0 (n_0 + K'/m)}{k_{10}\tau}\ , \tag{32}$$

$$\frac{dn_0}{d\upsilon_0{}'} = -\frac{\epsilon_0 n_0{}^2 (n_0 + K'/m)}{k_{10}\tau}, \tag{33}$$

$$\frac{dn_0}{dN_0} = -\frac{n_0{}^2 (K'/m)}{\tau}, \tag{34}$$

$$\frac{dn_0}{dk_{10}} = \frac{\epsilon_0 n_0 (\upsilon_0' n_0{}^2 + n_0 (K'/m\,\upsilon_0{}' + \upsilon_0) + K'/m\,\upsilon_0)}{\tau k_{10}{}^2}, \tag{35}$$

$$\frac{dn_0}{dV_A} = -\frac{2k_x{}^2 V_A (n_0 + 1)}{\tau k_{10}}\ , \tag{36}$$

where

$$\tau = 3n_0{}^2\left(1 + \frac{\epsilon_0 \upsilon_0{}'}{k_{10}}\right) + 2n_0\left(\frac{K'}{m} + \frac{\epsilon_0 \upsilon_0{}'}{k_{10}}\frac{K'}{m} + \frac{\epsilon_0 \upsilon_0}{k_{10}} + \frac{K' N_0}{\rho_0}\right) + \left(\frac{\epsilon_0 \upsilon_0}{k_{10}}\frac{K'}{m} + k_x{}^2 V_A{}^2 - \frac{g\beta k^2}{L}\right).$$

It is evident from equations (32)-(34) and (36) that the growth rates decrease with the increase of in kinematic viscosity, kinematic viscoelasticity, suspended particles number density and magnetic field if $\beta < 0$, so the system is stable.

If $\beta > 0$, then the growth rates decrease or increase with respect to viscosity, viscoelasticity, suspended particles number density and magnetic field according as

$$3n_0{}^2\left(1 + \frac{\epsilon_0 \upsilon_0{}'}{k_{10}}\right) + 2n_0\left(\frac{K'}{m} + \frac{\epsilon_0 \upsilon_0{}'}{k_{10}}\frac{K'}{m} + \frac{\epsilon_0 \upsilon_0}{k_{10}} + \frac{K' N_0}{\rho_0}\right) + \frac{\epsilon_0 \upsilon_0}{k_{10}}\frac{K'}{m} + k_x{}^2 V_A{}^2 \gtrless \frac{g\beta k^2}{L}.$$

From equation (35), the growth rates increase with the increase of medium permeability if $\beta < 0$, hence the system is unstable.

If $\beta > 0$, then growth rates increase or decrease with the increase of medium permeability according as

$$3n_0{}^2\left(1 + \frac{\epsilon_0 \upsilon_0{}'}{k_{10}}\right) + 2n_0\left(\frac{K'}{m} + \frac{\epsilon_0 \upsilon_0{}'}{k_{10}}\frac{K'}{m} + \frac{\epsilon_0 \upsilon_0}{k_{10}} + \frac{K' N_0}{\rho_0}\right) + \frac{\epsilon_0 \upsilon_0}{k_{10}}\frac{K'}{m} + k_x{}^2 V_A{}^2 \gtrless \frac{g\beta k^2}{L}.$$

CONCLUSION

The stability of incompressible Rivlin-Ericksen elastico-viscous superposed fluids permeated with suspended particles and variable horizontal magnetic field in porous medium has been investigated. For exponentially varying stratifications, it has been found that the system is stable for stable stratifications if $\beta < 0$ and unstable for unstable stratifications $\beta > 0$. The growth rates decrease rapidly with the increase of kinematic viscosity, kinematic viscoelasticity, suspended particles number density and magnetic field for low wave numbers and hence stabilize the system whereas medium permeability has destabilizing effect.

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